# **Geo Tech Note:**

## **Computation of Cross-Axis Tilts**

900 Series Biaxial Inclinometers



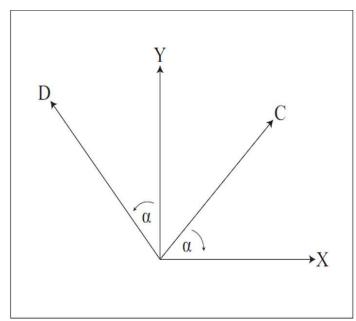


Figure 1. Plan View

### Assumptions

- 1. Planes XY and CD are initially horizontal.
- 2. Scale factors,  $S_\chi$  and  $S_\psi$  for rotations in vertical planes parallel to X and Y directions are given.
- 3. Voltage outputs of X and Y channels,  $V_X$  and  $V_Y$  are: measured voltages minus bias (see below).
- 4. Bias voltages, B<sub>x</sub> and B<sub>y</sub> have been measured. Bias voltages are the tiltmeter's X and Y outputs when it is perfectly horizontal.

#### **Problem**

- 1. Rotation occurs in a vertical plane parallel to direction C.
- 2. Find angle  $\alpha$ , which defines C direction.
- 3. Find amplitude of rotation,  $\theta_c$ .

#### Solution

For rotation in the X or Y directions, rotation amplitude is given by

 $\theta_{\rm x} = {\rm S_x \, V_x}$  and  $\theta_{\rm y} = {\rm S_y \, V_y}$  where  ${\rm S_x}$  and  ${\rm S_y}$  are the scale factors given in the user's manual or calibration certificate.

For rotation in the C direction we define new scale factors:

$$S_c' = S_x/\cos \alpha$$
 and  $S_c'' = S_x/\cos (90^\circ - \alpha)$ 

The rotation amplitude in the C direction then becomes:

$$\theta_{c} = S_{c}' V_{x} = (S_{x}/\cos \alpha) (V_{x})$$

[1]

$$\theta_C = S_C V_y = [S_y/\cos(90^\circ - \alpha)] (V_y)$$
 [

Equating [1] and [2] and solving gives:

$$\frac{S_{\chi}V_{\chi}}{\cos \alpha} = \frac{S_{\gamma}V_{\gamma}}{\cos (90^{\circ} - \alpha)}$$

$$\frac{S_{\chi}V_{\chi}}{S_{\nu}V_{\nu}} = \frac{\cos \alpha}{\cos (90^{\circ} - \alpha)} = \frac{1}{\tan \alpha}$$

$$\alpha = tan^{-1} S_{\gamma}V_{\gamma} S_{\chi}V_{\chi}$$
 [3]

Substituting  $\alpha$  in equation [1] or [2] along with the known values of  $S_x$  and  $V_{x'}$  or  $S_y$  and  $V_{y'}$  gives the rotation angle,  $\theta_C$ .

## Summary: Steps in Measuring Cross-Axis Tilt Angle, $\theta_c$

- 1. Measure bias voltages, B<sub>v</sub> and B<sub>v</sub>, on both tilt channels.
- 2. Subtract  $B_x$  and  $B_y$  from measured voltages to obtain corrected outputs  $V_v$  and  $V_v$  for use in eqns. [1] through [3].
- 3. Compute  $\alpha$  from equation [3] using known scale factors  $\boldsymbol{S}_\chi$  and  $\boldsymbol{S}_\gamma$
- 4. Compute  $\theta_c$  from equation [1] or [2].









