

Inertial Tech Note:

Non-Linearity Calculations



Non-linearity is the largest deviation in the sensors output over its specified input range when the output is compared to a least-squares-best-fit line.

In this example, we will look at the [LSRP](#) series of Inclinometer and a typical data set to calculate its non-linearity.



The LSRP Series of inclinometers are available with input ranges from $\pm 1.0^\circ$ to $\pm 90.0^\circ$, as shown in the table below from the [LSRP](#) series datasheet:

PERFORMANCE

INPUT RANGE (°)	± 1.0	± 3.0	± 14.5	± 30.0	± 90.0
FULL RANGE OUTPUT VDC (FRO $\pm 1\%$) ¹	± 5.0				
NON LINEARITY (%FRO max) ²	0.05	0.05	0.02	0.02	0.05
SCALE FACTOR (V/g, nominal)	286.5	95.5	20.0	10.0	5.0
SCALE FACTOR TEMP. SENSITIVITY (ppm/°C, max)	400	300	100	60	60
BANDWIDTH (-3db, Hz, nominal)	0.5	2.0	15.0	20.0	40.0
TRANSVERSE AXIS MISALIGNMENT (° max)	0.10	0.15	0.25	0.50	1.00
OUTPUT AT 0° TILT (Volts, max)	± 0.10	± 0.04	± 0.02	± 0.02	± 0.02
0° OUTPUT TEMP. SENSITIVITY (Volts/°C, max)	0.005	0.003	0.001	0.0005	0.0003
RESOLUTION & THRESHOLD (μ radian)	1				

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In this example we will look at the LSRP-90 model. It has a non-linearity specification of 0.05% FRO max*. The FRO of the sensor series is $\pm 5\text{VDC} \pm 1\%$; that is -5VDC at negative full input angle, -90° , to +5VDC at positive full input angle, $+90^\circ$. This implies that the output can potentially deviate as much as 0.005V or 5mV. But how is this calculated?

Every sensor unit comes with an accompanying test report published and provided to the customer. Included in this report is a linearity data set generated by Jewell's manufacturing team during final evaluation prior to shipping. The data was generated by measuring the output of the LSRP unit at known tilt angles across the operating range of the sensor, using calibrated test stands. The table below displays the following raw experimental data:

- Input angle ($^\circ$) this is the expected angular position of each test.
- Measured Output (VDC) this is the VDC output of the sensor at each tested angular position. This data is then processed to generate the non-linearity result.

Linearity Data X-Axis

Input Angle ($^\circ$)	Measured Output (VDC)	Calculated Output (VDC)	Calculated Deviation	Error (% FRO)
-90.000	-5.0178	-5.0189	0.0011	0.011
-72.000	-4.7708	-4.7710	0.0002	0.002
-54.000	-4.0573	-4.0567	-0.0005	-0.005
-36.000	-2.9469	-2.9460	-0.0009	-0.009
-18.000	-1.5483	-1.5475	-0.0007	-0.007
0.000	0.0016	0.0018	-0.0001	-0.001
18.000	1.5507	1.5502	0.0005	0.005
36.000	2.9471	2.9463	0.0008	0.008
54.000	4.0539	4.0533	0.0006	0.006
72.000	4.7629	4.7629	-0.0000	-0.000
90.000	5.0047	5.0056	-0.0009	-0.009

Standard Error X-Axis % FRO = 0.007

The "Standard Error X-Axis % FRO" result of 0.007 is the result of a non-linearity calculation performed using the dataset above. The following steps will describe how this is calculated using the "least-squares-best-fit line" method.

- Firstly, the test angle is adjusted by adding the units output axis misalignment (O_{am}) from the test angle. Prior to this test O_{am} was calculated to be 0.096° . For example $-90^\circ + 0.096^\circ = -89.904$. The table below shows the "corrected" angle for each test position.

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Test Angle	Corrected Angle
-90	-89.904
-72	-71.904
-54	-53.904
-36	-35.904
-18	-17.904
0	0.096
18	18.096
36	36.096
54	54.096
72	72.096
90	90.096

- The corrected angle is then used to calculate the output as acceleration - scaled to g's (9.81m/s²) - this is simply done by calculating the sin value of each corrected angle in °:

Test Angle	Corrected Angle	Measured Output Voltage (VDC)	Acceleration (g's)
-90	-89.904	-5.0178	-1.0000
-72	-71.904	-4.7708	-0.9505
-54	-53.904	-4.0573	-0.8080
-36	-35.904	-2.9469	-0.5864
-18	-17.904	-1.5483	-0.3074
0	0.096	0.0016	0.0017
18	18.096	1.5507	0.3106
36	36.096	2.9471	0.5891
54	54.096	4.0539	0.8100
72	72.096	4.7629	0.9516
90	90.096	5.0047	1.0000

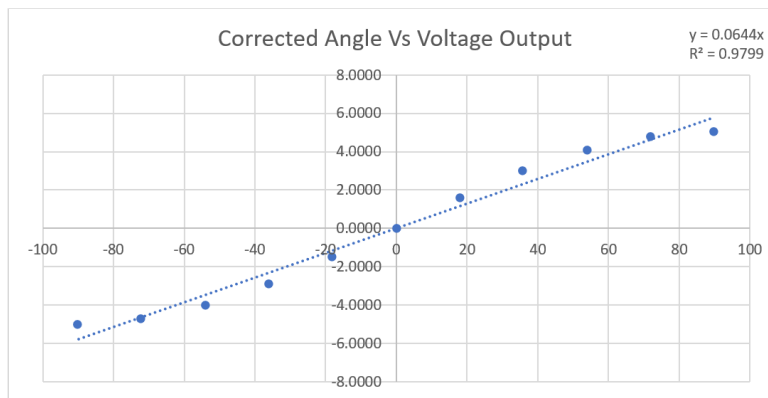
- The Scale factor is then calculated by calculating the slope of the measured output Voltage vs Acceleration in g's calculated in the previous step. The result is a scale factor of 5.0123 V/g. Note that this calculation can be done using the SLOPE function in Microsoft Excel.
- We then calculate the Intercept of the same Output Voltage (VDC) vs Acceleration, which results in an intercept of -0.0067.
- The Intercept and Scale factor is then used to calculate the expected voltage output of the sensor by multiplying the Calculated Output (VDC) by the Scale factor (V/g) and adding the intercept.

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- The output deviation is then calculated by subtracting the calculated Voltage output (VDC) from the Measured Voltage output (VDC).
- The Full range output (FRO, V) is then calculated by taking the Voltage range IE the calculated voltage output at -90° minus the calculated output at $+90^\circ$. This results in an FRO of 10.0246V.
- We then calculate % non-linearity at each angle by dividing the output deviation at each angle by the FRO:



- Note that the non-linearity in the units test report is the standard deviation of the % deviations across the full range. This turns out to be 0.0065 % FRO, which rounds up to the 0.007% FRO reported in the test report.

Test Angle	Corrected Angle	Measured Output Voltage (VDC)	Acceleration (g's)	Calculated Output (VDC)	Output Deviation (VDC)	Non linearity % FRO (Full Range Output)
-90	-89.904	-5.0178	-1.0000	-5.0189	0.0011	0.0114
-72	-71.904	-4.7708	-0.9505	-4.7710	-0.0002	-0.0023
-54	-53.904	-4.0573	-0.8080	-4.0567	0.0006	0.0055
-36	-35.904	-2.9469	-0.5864	-2.9460	0.0009	0.0089
-18	-17.904	-1.5483	-0.3074	-1.5476	0.0007	0.0075
0	0.096	0.0016	0.0017	0.0017	0.0001	0.0014
18	18.096	1.5507	0.3106	1.5502	-0.0005	-0.0049
36	36.096	2.9471	0.5891	2.9463	-0.0008	-0.0081
54	54.096	4.0539	0.8100	4.0533	-0.0006	-0.0059
72	72.096	4.7629	0.9516	4.7629	0.0000	0.0001
90	90.096	5.0047	1.0000	5.0056	0.0009	0.0093
					STD of Non linearity	0.0065
Scale Factor (V/g)		5.0123				
Intercept		-0.0067				
FRO (Full Range Output)		10.0246				
Output Axis misalignment °		0.096				

* Non-linearity is specified as deviation of output referenced to theoretical sine function value, independent of misalignment